Stand density index in uneven-aged ponderosa pine stands

C.W. Woodall, C.E. Fiedler, and K.S. Milner

Abstract: Stand density index (SDI) was developed to quantify relative stand density in even-aged stands. Application of SDI in uneven-aged stands has been described mathematically but not justified biologically. Diameter-class trends in SDI and sapwood area across 14 uneven-aged ponderosa pine (*Pinus ponderosa* Dougl. ex P. & C. Laws.) stands in eastern Montana were examined to elucidate the biological underpinnings of the SDI summation method. Results indicate that the SDI summation method is biased in its apportionment of relative stand density across diameter classes in uneven-aged ponderosa pine stands. SDI apportions greater relative density to small trees than to larger ones. Therefore, SDI may overpredict site occupancy for reverse J-shaped diameter distributions with more small trees than large ones, and it may underpredict occupancy with nonreverse J-shaped diameter distributions. Application of the SDI summation method in uneven-aged ponderosa pine stands may be biologically justified only if site occupancy – diameter-class trends are taken into account when interpreting SDI values. Replacing the self-thinning scaling factor of the SDI summation method with more biologically relevant scaling relationships may create improved relative density measures for uneven-aged stands.

Résumé : L'indice de densité de peuplement (IDP) a été développé pour quantifier la densité relative des peuplements équiennes. L'application de l'IDP dans les peuplements inéquiennes a été décrite mathématiquement, mais elle n'a pas été justifiée sur le plan biologique. Pour élucider les fondements biologiques de la méthode de sommation des IDP, le comportement de l'IDP et de la surface d'aubier en fonction des classes de diamètre a été examiné dans 14 peuplements inéquiennes de pin ponderosa (*Pinus ponderosa* Dougl. ex P. & C. Laws.). Les résultats indiquent que la méthode de sommation des IDP alloue de façon biaisée la densité relative du peuplement aux diverses classes de diamètre dans les peuplements inéquiennes de pin ponderosa. La méthode de sommation des IDP alloue la densité relative de façon plus importante aux petits arbres qu'aux grands arbres. Par conséquent, la méthode de sommation des IDP peut surestimer l'occupation du site par les petits arbres lorsque la distribution des diamètres suit la courbe en J inversé et la sous-estimer lorsque la distribution des diamètres suit la courbe en J non inversé. L'application de la méthode de sommation des IDP dans les peuplements inéquiennes de pin ponderosa peut être biologiquement justifiée seulement si l'occupation du site par les différentes classes de diamètre est prise en compte pour interpréter les valeurs de l'IDP. Le remplacement du facteur d'échelle de l'auto-éclaircie dans la méthode de sommation des IDP par des relations d'échelle biologiquement plus pertinentes peut permettre d'améliorer les mesures de la densité relative des peuplements inéquiennes.

[Traduit par la Rédaction]

Introduction

Stand density index (SDI; Reineke 1933) is a relative density measure that was developed for even-aged stands but has been applied in uneven-aged stands. SDI is based on the relationship between mean tree size and number of trees per unit area in a forest stand. The diameter class summation method is used to apply SDI in uneven-aged stands, where SDI values are calculated for each diameter class and then summed for an estimate of total stand density (Stage 1968; Long 1995). The diameter class summation method has been described mathematically (Shaw 2000), but a biological justification has never been demonstrated for uneven-aged stands. The goal of this study is to evaluate the biological basis for applying the SDI summation method in uneven-aged stands by examining SDI and sapwood area values across diameter classes and among stands in uneven-aged forests.

Received 7 March 2002. Accepted 28 October 2002. Published on the NRC Research Press Web site at http://cjfr.nrc.ca on 20 December 2002.

C.W. Woodall.[1] USDA Forest Service, North Central Research Station, St. Paul, MN 55108, U.S.A.
C.E. Fiedler and K.S. Milner. University of Montana, Missoula, MT 59812, U.S.A.

[1]Corresponding author (e-mail: cwoodall@fs.fed.us).

Past work

SDI in even-aged stands

Many indices of growing stock, such as SDI, incorporate measures of mean tree size and density (Long and Smith 1984; Long 1985, 1996). SDI, first proposed by Reineke (1933) as a measure of stocking in even-aged stands, is based on the premise that as mean plant size per unit area increases, the number of individuals per unit area decreases (Enquist et al. 1998). SDI is defined as the equivalent trees per hectare at a quadratic mean diameter of 25 cm and is formulated as

$$\text{[1]}\qquad \mathrm{SDI} = \mathrm{TPH}\left(\frac{\mathrm{DBH_q}}{25}\right)^{1.6}$$

 doi: 10.1139/X02-168

where SDI is stand density index, TPH is number of trees per hectare, and DBH_q is quadratic mean diameter (cm) at breast height (1.4 m) (Long 1985).

SDI in uneven-aged stands

Size–density relationships form the conceptual foundations of SDI. However, inherent attributes of uneven-aged stands complicate the application of size–density theories. First, because of the wide range of tree sizes in uneven-aged stands, mean tree size descriptors may vary substantially (Oliver and Larson 1996). Second, common stand descriptors such as TPH and mean tree size may reveal little about uneven-aged stands (Fiedler and Cully 1995; Shaw 2000). Uneven-aged stands can have approximately the same TPH regardless of mean diameter (Sterba and Monserud 1993). Third, uneven-aged stands in a broader sense appear to violate the size–density formulations, because these stands have variable levels of competition for individual trees (Burton 1993).

Stage (1968) addressed some of the factors confounding the application of SDI in uneven-aged stands by proposing the summation method, which describes the relative contribution of each diameter class to the overall stand SDI. The summation method calculates SDI for each diameter class and then sums for a total stand value (Stage 1968). The SDI summation method formulation currently used by foresters and researchers alike is

$$[2] \qquad \mathrm{SDI} = \sum \mathrm{TPH}_i \left(\frac{\mathrm{DBH}_i}{25}\right)^{1.6}$$

where DBH_i is the midpoint of the ith diameter class (cm) and TPH_i is the number of trees per hectare in the ith diameter class (Long 1995; Shaw 2000).

SDI, using the diameter class summation method, was proposed as a relative density measure for uneven-aged stands in a number of studies (Long and Daniel 1990; Long 1995; Fiedler and Cully 1995). While the formulation of the SDI summation method is mathematically derived (Stage 1968; Shaw 2000), a biological basis for its application in uneven-aged stands has not been demonstrated. The SDI summation method implicitly assumes a constant SDI – site occupancy (total stand sapwood area) relationship across a range of diameters. However, application of SDI in uneven-aged stands may require information on actual site occupancy relative to SDI values across ranges of diameters, which may ultimately provide more accurate stand-level density estimates.

Objectives

The goal of this investigation was to evaluate the biological basis for applying the SDI summation method in uneven-aged stands. Assuming individual tree sapwood area may closely reflect the actual site occupancy of any given tree (pipe model theory; Shinozaki et al. 1964; Waring et al. 1982), an examination of SDI values and sapwood areas for diameter classes and entire stands was conducted in uneven-aged stands to (*i*) compare SDI per hectare and sapwood area per hectare for uneven-aged stands, (*ii*) examine trends in sapwood area versus DBH across a range of tree diameters, (*iii*) examine trends in SDI versus sapwood area across diameter classes, and (*iv*) develop recommendations for SDI application in uneven-aged stands based on evaluation of the diameter class summation method and sapwood area trends.

Study area

Ponderosa pine (*Pinus ponderosa* Dougl. ex P. & C. Laws.) occupies approximately 0.9×10^6 ha in eastern Montana, often in monocultures (O'Brien and Conner 1991; O'Brien and Collins 1991). Ponderosa pine forests occur on lowlands and hilly terrain (850–1350 m in elevation) in the northern Great Plains of eastern Montana (Arno 1979), where soils are typically shallow and poorly developed, and precipitation averages 26–42 cm a year (Pfister et al. 1977). Because of the historic low-intensity fire regimes, episodic regeneration events, and harsh environmental conditions of eastern Montana, many of these stands are uneven aged.

Methods

Field

Fourteen pure, uneven-aged ponderosa pine stands were located over a wide range of site quality. Stands were characterized as fully stocked if they met certain criteria: sparse regeneration, a substantial number of suppressed trees, and few canopy gaps. Stands characterized as understocked had the following attributes: vigorous regeneration, sufficient canopy gaps to allow for mid-canopy crown expansion, and no self-thinning or suppression. Six understocked and eight fully stocked stands were sampled. These stands exhibited negligible evidence of human or recent natural disturbances and were deemed uneven aged by preliminary coring of various sizes of trees.

One 0.2- to 0.4-ha fixed-radius plot was subjectively established at each site to minimize variation in physiographic attributes. Plot trees between 3.8 and 12.7 cm DBH were cored once to the pith, and plot trees ≥12.7 cm in DBH were cored twice to the pith (upslope or downslope then sideslope). Once a core was extracted, its sapwood–heartwood boundary was marked with a pen, and the core was inserted into a labeled plastic straw for future analysis in the laboratory. In addition to core work, all trees were measured for DBH. Approximately 5000 trees were sampled across the 14 sites (210–605 trees per site).

Laboratory procedures and data tabulation

Every core taken in the field was dried, mounted on a grooved board, and sanded for subsequent analysis. Sapwood width and inside-bark bole radius were measured to the nearest millimetre. Inside bark basal area and sapwood area were calculated for each tree, averaging dual estimates for trees that were cored twice. SDI was calculated for each individual tree using the SDI summation method (eq. 2).

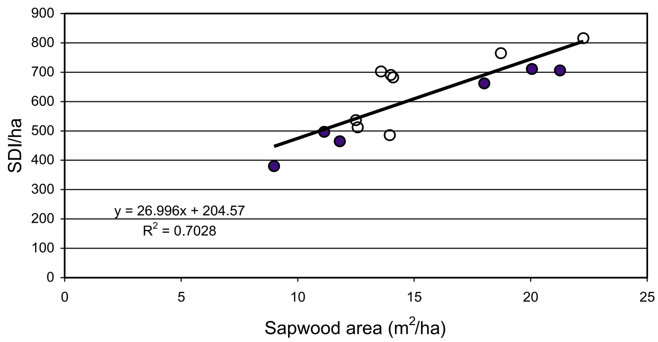
Data analysis

The relationships between stand SDI and sapwood area occupancy:

$$[3] \qquad E(\mathrm{SDI}) = b_0 + b_1(\text{stand sapwood area})$$

and between sapwood area occupancy and individual diameters:

Fig. 1. SDI per hectare versus sapwood area per hectare for uneven-aged ponderosa pine stands in eastern Montana. Open symbols are reverse J-shaped diameter distributions, and solid symbols are non-reverse J-shaped diameter distributions.



$$[4] \qquad E(\text{SDI}) = b_0 + b_1(\text{DBH-class sapwood area})$$

were estimated using linear regression. The trend in DBH versus sapwood area for individual trees was estimated using nonlinear regression analysis:

$$[5] \qquad E(\text{individual tree sapwood area}) = b_0 + (\text{DBH})^{b_1}$$

where E is the expected value and b_0 and b_1 are parameters to be estimated.

Results and discussion

For all sites, per-hectare SDI values calculated using the summation method had a positive correlation with estimates of stand sapwood area per hectare ($p < 0.01$) (Fig. 1). Although stand SDI increased with increasing levels of sapwood area, considerable residual variation remained about the regression line (eq. 3) ($R^2 = 0.70$, SE ($\hat{b}_0$) = 79.62, SE ($\hat{b}_1$) = 5.07) (Fig. 1). Assuming total sapwood area reflects leaf area in the stand (Waring et al. 1982), SDI per hectare values may not be apportioning site occupancy across diameter distributions so as to directly reflect total stand site occupancy. The residual variation about the regression line may be attributable to the particular diameter distribution of each sample stand. The majority of stands with reverse J-shaped diameter distributions have relatively higher SDI values per unit of stand sapwood area, while the majority of stands with non-reverse J-shaped diameter distributions (those with relatively few small trees) appear to have lower SDI values per unit of sapwood area (Fig. 1).

Since the diameter distributions of individual uneven-aged stands may be affecting the relationship between SDI and actual levels of sapwood area, trends in SDI versus sapwood area across diameter classes were examined. The SDI method uses the value $\text{DBH}^{1.6}$ to weight the relative contribution of an individual tree to overall site occupancy. The question arises: is this also the power to which DBH should be raised to reflect sapwood area trends across diameter classes in uneven-aged ponderosa pine stands? An estimate of this value (1.76) was obtained using non-linear regression (eq. 5) (n = 5032, SE ($\hat{b}_0$) = 1.1292, SE ($\hat{b}_1$) = 0.0013). Considering the $\hat{b}_1$ value and the associated standard error for the previous model, the exponent value (1.6) of the SDI method is not the same as the exponent value (1.76) of sapwood area. Previous work suggests a possible relationship

Fig. 2. Percentage of total stand value of SDI (DBH) and sapwood area (DBH) for a flat diameter distribution with one tree per each 5-cm diameter class based on sapwood area – DBH relationships in uneven-aged ponderosa pine stands in eastern Montana.

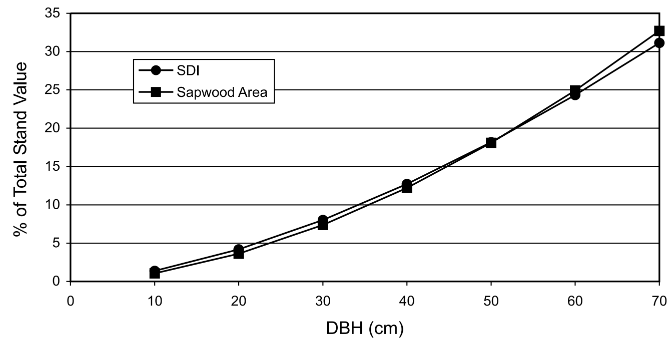
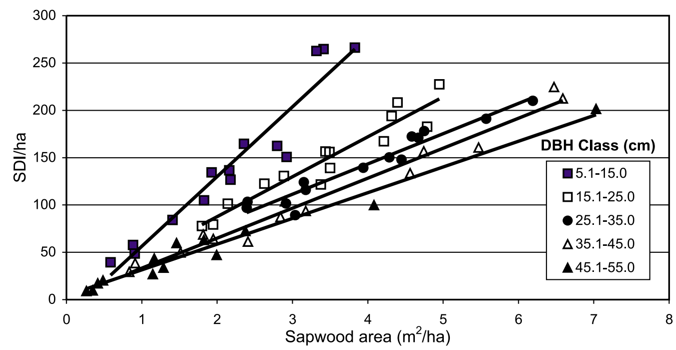


Fig. 3. SDI per hectare versus sapwood area per hectare for 10-cm DBH classes in uneven-aged ponderosa pine stands in eastern Montana.



between SDI and sapwood area or leaf area values but does not establish it (Long and Daniel 1990; Long and Smith 1984). When trends in SDI and sapwood area across diameter classes are considered in a cumulative manner, as they are when using the SDI summation method, definitive patterns emerge (Fig. 2). The power of 1.76 associated with the sapwood area equation serves to apportion relatively more value to large-diameter than small-diameter trees compared with the 1.6 power of SDI (Fig. 2). When taken cumulatively, the slight differences between actual site occupancy (sapwood area) and SDI for individual trees may lead to significant differences in actual site occupancy and SDI for diameter classes and entire stands (Figs. 1 and 2).

SDI – sapwood area trends across diameter distributions were further estimated using regression analyses to evaluate the dependence of SDI on sapwood area levels for 10-cm diameter classes (eq. 4). Regardless of diameter class, there appears a positive linear relationship of increasing SDI values with increasing sapwood area values for each diameter class (Fig. 3). However, model slope estimates ($\hat{b}_1$) indicate that as diameter increases, the predicted values of SDI per unit sapwood area decreases (Table 1). Differences in slope estimates diminish as diameter increases (Table 1), with an F test ($\alpha = 0.05$) indicating significant differences between slope parameter estimates for all successive DBH classes except the 15.1- to 25- and 25.1- to 35-cm DBH classes (extra sum of squares principle; Ratkowsky 1983). The dispropor-

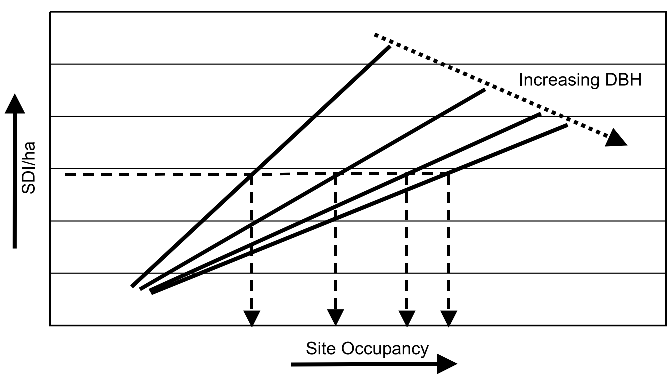
Table 1. Linear regression output for model: SDI/ha = b_0 + b_1(sapwood area, m^2/ha).

Diameter class (cm)	n	R^2	Parameter	Parameter estimate	Residual SE	Significance $P >\|t\|$
5.1–15.0	14	0.92	b_0	–17.568	14.739	0.2563
			b_1	73.775	6.181	<0.0001
15.1–25.0	14	0.91	b_0	3.136	13.507	0.8203
			b_1	42.230	3.792	<0.0001
25.1–35.0	14	0.94	b_0	15.381	9.779	0.1417
			b_1	31.997	2.374	<0.0001
35.1–45.0	14	0.97	b_0	1.612	5.719	0.7828
			b_1	31.663	1.506	<0.0001
45.1–55.0	13	0.97	b_0	4.291	3.671	0.2671
			b_1	27.162	1.426	<0.0001

tionate allocation of SDI to basal area of small-diameter trees was first described by Stage (1968) in grand fir (*Abies grandis* (Dougl.) Lindl.) stands but in relation to growth. Reverse J-shaped diameter distributions, which contain more small-diameter trees than larger ones, would be expected to have higher SDI values than actual site occupancy values, an assumption supported by Figs. 1–3. In stands with approximately equal numbers of trees in all diameter classes, or more medium- to large-sized trees than small-sized trees (i.e., non-reverse J-shaped diameter distributions), SDI would be expected to underpredict actual site occupancy (Fig. 2).

These results are based on the assumption that sapwood area is an adequate surrogate for site occupancy (leaf area) across all diameter classes. However, this assumption may not hold if leaf area/sapwood area ratios vary by tree diameter or site quality. Past research suggests that the leaf area of larger trees may be growth inefficient because of decreases in sapwood permeability, hydraulic conductance, and whole-tree conductance (Whitehead et al. 1984; Ryan and Yoder 1997; Ryan et al. 2000). Some research has suggested that these inefficiencies may result in a lower leaf area/sapwood area ratio for increasing tree sizes (Whitehead et al. 1984), while a study specific to uneven-aged ponderosa pine found no difference in leaf area/sapwood area prediction equations among trees in different canopy strata (O'Hara and Valappil 1995). The possible effects of site quality on leaf area/sapwood area ratios currently remain undetermined, with research indicating no site effect for loblolly pine (*Pinus taeda* L.) (Shelburne et al. 1993) and possible regional differences between equations for Rocky Mountain and Pacific Northwest ponderosa pine (O'Hara and Valappil 1995). Although quantification of potential bias in sapwood area/leaf area ratios has neither been clearly substantiated nor rejected, the application of the pipe model theory to silvicultural problems has led to substantial progress in managing stands on a more physiological basis (Margolis et al. 1995). Given this caveat, the results of our study suggest that strict application of the SDI summation method in uneven-aged ponderosa pine stands cannot be justified biologically. The SDI summation method should be applied in uneven-aged stands only if site occupancy – diameter-class trends are taken into account when interpreting SDI values (Fig. 4). Assuming that sapwood area is an adequate surrogate for actual site occupancy, the site occupancy for a given SDI value will be expected to increase as tree diameter increases (Fig. 4). However, this rate of increase will diminish with increasing diameter (Fig. 4). Conceptualization of site occupancy – SDI diameter-class trends may provide a more robust method of applying SDI in uneven-aged stands. Possibly the SDI formulation itself may be modified. The biological significance of the 1.6 self-thinning power (Reineke 1933) has been questioned previously (Weller 1987; Enquist et al. 1998) and may be an inappropriate basis for relative density measures in uneven-aged stands. However, the SDI concept of weighting the contribution of each tree to overall site occupancy may provide a framework for developing relative density measures for uneven-aged stands, although biological phenomena other than self-thinning may provide a more appropriate weighting scheme for individual trees. Power scaling functions (Enquist et al. 2000), such as the sapwood = $DBH^{1.76}$ power function determined in this study, could be incorporated into new relative density formulations.

Fig. 4. Theoretical relationship between SDI and actual site occupancy for individual diameter classes.



Conclusions

The summation method of calculating SDI is biased in its apportionment of site occupancy across diameter classes in ponderosa pine stands. The amount of SDI apportioned to each diameter class, relative to the actual sapwood area in that class, declines as diameter increases. Therefore, SDI will overpredict site occupancy with reverse J-shaped diameter distributions and underpredict occupancy with flat or other non-reverse J-shaped diameter distributions. The SDI summation method should only be applied in uneven-aged stands, if one is cognizant of the effect of diameter distributions on SDI interpretation. Combining the SDI summation methodology with allometric scaling alternatives to the self-thinning principle may form a basis for developing more robust and biologically relevant relative density measures for uneven-aged stands.

Acknowledgments

This study was funded by a grant from the McIntire-Stennis program at the University of Montana. The authors thank Mark Hansen, Kathleen Kavanagh, Ron McRoberts, and two anonymous reviewers for valuable comments on earlier versions of this manuscript. This study was facilitated by the hard fieldwork of Todd Morgan and Michael Heaney.

References

Arno, S.F. 1979. Forest regions of Montana. USDA For. Serv. Res. Pap. INT-218.

Burton, P.J. 1993. Some limitations inherent to static indices of plant competition. Can. J. For. Res. **23**: 2141–2152.

Enquist, B.J., Brown, J.H., and West, G.B. 1998. Allometric scaling of plant energetics and population density. Nature (London), **395**: 163–165.

Enquist, B.J., West, G.B., and Brown, J.H. 2000. Quarter-power allometric scaling in vascular plants: functional basis and ecological consequences. *In* Scaling in biology. *Edited by* J.H. Brown and G.B. West. Oxford University Press, New York. pp 167–198.

Fiedler, C.E., and Cully, J.E. 1995. A silvicultural approach to develop Mexican spotted owl habitat in Southwest forests. West. J. Appl. For. **10**: 144–148.

Long, J.N. 1985. A practical approach to density management. For. Chron. **61**: 23–27.

Long, J.N. 1995. Using stand density index to regulate stocking in uneven-aged stands. *In* Uneven-aged management — opportunities, constraints, and methodologies. *Edited by* K. O'Hara. Montana Forest and Conservation Experiment Station, Missoula, Mont. Misc. Publ. 56. pp. 110–122.

Long, J.N. 1996. A technique for the control of stocking in two-storied stands. West. J. Appl. For. **11**: 59–61.

Long, J.N., and Daniel, T.W. 1990. Assessment of growing stock in uneven-aged stands. West. J. Appl. For. **5**: 93–96.

Long, J.N., and Smith, F.W. 1984. Relation between size and density in developing stands: a description and possible mechanism. For. Ecol. Manage. **7**: 191–206.

Margolis, H., Oren, R., Whitehead, D., and Kaufmann, M.R. 1995. Leaf area dynamics of conifer forests. *In* Ecophysiology of coniferous forests. *Edited by* W.K Smith and T.H. Hinckley. Academic Press, New York. pp. 181–223.

O'Brien, R.A., and Collins, D.C. 1991. Forest statistics for land outside National Forests in west-central Montana. USDA For. Serv. Res. Bull. INT-72.

O'Brien, R.A., and Conner, R.C. 1991. Forest statistics for land outside National Forests in eastern Montana. USDA For. Serv. Res. Bull. INT-71.

O'Hara, K.L., and Valappil, N.I. 1995. Sapwood–leaf area prediction equations for multi-aged ponderosa pine stands in western Montana and central Oregon. Can. J. For. Res. **25**: 1553–1557.

Oliver, C.D., and Larson, B.C. 1996. Forest stand dynamics. Update edition. John Wiley & Sons, New York.

Pfister, R.D., Kovalchik, B.L., Arno, S.F., and Presby, R.C. 1977. Forest habitat types of Montana. USDA For. Serv. Gen. Tech. Rep. INT-34.

Ratkowsky, D.A. 1983. Nonlinear regression modeling. Marcel Dekker, Inc., New York.

Reineke, L.H. 1933. Perfecting a stand-density index for even-aged stands. J. Agric. Res. **46**: 627–638.

Ryan, M.G., and Yoder, B.J. 1997. Hydraulic limits to tree height and tree growth — what keeps trees from growing beyond a certain height? BioScience, **47**: 235–242.

Ryan, M.G., Bond, B.J., Law, B.E., Hubbard, R.M., Woodruff, D., Cienciala, E., and Kucera, J. 2000. Transpiration and whole-tree conductance in ponderosa pine trees of different heights. Oecologia, **124**: 553–560.

Shaw, J.D. 2000. Application of stand density index to irregularly structured stands. West. J. Appl. For. **15**: 40–42.

Shelburne, V.B., Hedden, R.L., and Allen, R.M. 1993. The effects of site, stand density, and sapwood permeability on the relationship between leaf area and sapwood area in loblolly pine (*Pinus taeda* L.). For. Ecol. Manage. **58**: 193–209.

Shinozaki, K., Yoda, K., Hozumi, K., and Kira, T. 1964. A quantitative analysis of plant form — the pipe model theory. Jpn. J. Ecol. **14**: 97–104.

Stage, A.R. 1968. A tree-by-tree measure of site utilization for grand fir related to stand density index. USDA For. Serv. Res. Note. INT-77.

Sterba, H., and Monserud, R.A. 1993. The maximum density concept applied to uneven-aged mixed-species stands. For. Sci. **39**: 432–452.

Waring, R.H., Schroeder, P.E., and Oren, R. 1982. Application of the pipe model theory to predict canopy leaf area. Can. J. For. Res. **12**: 556–560.

Weller, D. 1987. A re-evaluation of the –3/2 power rule of plant self-thinning. Ecol. Monogr. **57**: 23–43.

Whitehead, D., Edwards, W.R.N., and Jarvis, P.G. 1984. Conducting sapwood area, foliage area, and permeability in mature trees of *Picea sitchensis* and *Pinus contorta*. Can. J. For. Res. **14**: 940–947.